Supplemental Materials for Robert Innes and Joseph Bial, "Inducing Innovation in the Environmental Technology of Oligopolistic Firms," Journal of Industrial Economics VOLUME 50 (ISSUE 3), September 2002, pp. 265-288.

A. Proofs of Section V Results

First note that, for simplicity, we ensure interior emission choices by assuming that, for $\delta \in \{0,1\}$, $|c_s(0,\delta)|$ is arbitrarily large and $|c_s(s,\delta)| \approx 0$ for all s above a given (positive) $\bar{s}$. The bound $\bar{s}$ represents a "zero abatement" level of per-unit emissions and is thus assumed to satisfy: $c(\bar{s},1) \approx c(\bar{s},0)$. Second, we establish a sequence of preliminary results.

Result I. (a) $t_e(s^*(1)) < t_1$, and (b) $t_e(s^*(0) < t_0$.

Proof. (a) Follows from eq. (14). (b) From (21),

$t_e(s^*(0)) = [C_1^* - c(s^*(0),0)]/s^*(0) = \{[C_1^* - C_0^*]/s^*(0)\} + t_0 < t_0$,

where the inequality is due to $C_1^* < C_0^*$.

Result II. $s^*(1) < s(t_e(s^*(1)),0)$, where

(A1) $$s(t,\delta) \equiv \text{argmin } c(s,\delta)+ts$$

Proof. Follows from Result I(a), $s_\delta() < 0$, $s_t() < 0$, and $s^*(1) = s(t_1,1)$.

Result III. $c_s(s^*(1),0) + t_e(s^*(1)) < 0$.

Proof. Follows from Result II, the definition of $s(t,\delta)$ in (A1) (where $c_s(s(),\delta)+t=0$), and $c_{ss} > 0$.

Result IV. $c_s(\bar{s},0) + t_e(\bar{s}) > 0$.

Proof. With $c_s(\bar{s},0) \approx 0$, $c(s^*(1),1) > c(\bar{s},1) \approx c(\bar{s},0)$, and $t_1 s^*(1) > 0$, we have (using (21))

$$c_s(\bar{s},0) + t_e(\bar{s}) \approx t_e(\bar{s}) = \{c(s^*(1),1) + t_1 s^*(1) - c(\bar{s},0)\}/\bar{s} > 0.$$

Result V. There is a unique $s_L^0 \in (s^*(1),\bar{s})$ such that (a) $c_s(s_L^0,0) + t_e(s_L^0) = 0$, (b) $c_s(s_L,0) + t_e(s_L) < 0$ for all $s_L < s_L^0$, and (c) $c_s(s_L,0) + t_e(s_L) \geq 0$ for all $s_L \geq s_L^0$.

Proof. By Results III and IV, and the Intermediate Value Theorem, there exists an $s_L^1 \in (s^*(1),\bar{s}): c_s(s_L^1,0)+t_e(s_L^1)=0$. We can thus define s_L^0 = min s_L: $s_L>s^*(1)$ and $c_s(s_L,0)+t_e(s_L)=0$. By construction, this s_L^0 satisfies properties (a) and (b). Now suppose that s_L^0 did not satisfy property (c), so that there is an $s_L^2 > s_L^0 : c_s(s_L^2,0)+t_e(s_L^2)<0$. By differentiability of $(c_s(s_L,0)+t_e(s_L))$, there must then exist an $s_L^3 \in [s_L^0, s_L^2)$: $c_s(s_L^3,0)+t_e(s_L^3)=0$ and $c_{ss}(s_L^3,0)+[\partial t_e(s_L^3)/\partial s_L]<0$. However, with $c_{ss}()>0$ and (from (21)),

(A2) $$\{c_s(s_L,0)+t_e(s_L)\} \overset{s}{=} - dt_e(s_L)/ds_L,$$

$c_{ss}()+[\partial t_e()/\partial s_L]>0$ whenever $c_s()+t_e()=0$, thus contradicting our premise that property (c) did not hold. Together, properties (a)-(c) imply uniqueness of s_L^0.

<u>*Result VI*</u>. $s^*(0)<s_L^0$ (with s_L^0 defined in Result V) .

Proof. From the definitions of $s^*(0)$, t_0, and $s(t,\delta)$ (in (A1))

(A3) $$s^*(0)=s(t_0,0) < s(t_e(s^*(0)),0),$$

where the inequality follows from Result I(b) and $s_t()<0$. (A3) further implies (using the definition of $s(t,\delta)$ and $c_{ss}>0$)

(A4) $$c_s(s^*(0),0)+t_e(s^*(0)) < 0.$$

Result VI now follows from (A4) and Result V.

<u>*Result VII*</u>. $c_s(s_L,0)+t_e(s_L)<0$ for all $s_L \in [s^*(1),s^*(0)]$.

Proof. Follows from Results V and VI.

<u>*Result VIII*</u>. $t_e(s^*(0))<t_1$.

Proof. From eq. (21),

(A5) $$t_e(s^*(0)) = [C_1^* - c(s^*(0),0)]/s^*(0) < [c(s^*(0),1)+t_1 s^*(0)-c(s^*(0),0)]/s^*(0)$$
$$= t_1 + [c(s^*(0),1)-c(s^*(0),0)]/s^*(0) < t_1,$$

where the first inequality is due to the definition of C_1^* =min $c(s,1)+t_1 s$, and the final inequality is due to $c_\delta<0$.

<u>*Result IX*</u>. For $s_L \in [s^*(1),s^*(0)]$, $dt_e(s_L)/ds_L>0$.

Proof. Follows from Result VII and (A2).

<u>*Proof of Observation 1*</u>. Follows from Results I(b), VIII, and IX. *QED*.

Proof of Corollary 1. With $t_e(s_L)<t_0$ (Observation 1) and $s_t()<0$, we have: $s(t_e(s_L),0) > s(t_0,0) = s^*(0) \geq s_L$. Similarly, with $t_e(s_L)<t_1$ (Observation 1), we have: $s(t_e(s_L),1) > s(t_1,1) = s^*(1) = s_W$. *QED*.

Proof of Observation 3. If

$$\Delta \equiv [W^*(1)-W^*(0)] - \pi_B(s^*(0)) > 0 \tag{A6}$$

then the observation follows from eq. (20) (where $\pi_B \equiv \pi_B(s^*(1))$) and the Intermediate Value Theorem. Expanding $\pi_B(s^*(0))$ from (22), we have

$$\pi_B(s^*(0)) = (C_0^* - C_1^*)Q^*(1) + \{t_1 s^*(1) - t_0 s^*(0) + [C_1^* - c(s^*(0),0)][1-(s^*(1)/s_L)]\}. \tag{A7}$$

Substituting (A7) into (A6), using eq. (18), and recalling that $t_\delta = D'(E^*(\delta))$ and $E^*(\delta) = s^*(\delta)Q^*(\delta)$, we have

$$\Delta = X + t_0 s^*(0)(Q^*(1)-Q^*(0)) + [D(E^*(0))-D(E^*(1))] - [C_1^* - c(s^*(0),0)][1-(s^*(1)/s_L)]Q^*(1), \tag{A8}$$

with (recalling Figure 1, where X corresponds with the negative of area c)

$$X \equiv \int_{Q(C_0^*)}^{Q(C_1^*)} (P(Q)-C_0^*)\, dQ > (C_1^* - C_0^*)(Q^*(1)-Q^*(0)). \tag{A9}$$

Using (A9), and substituting $t_e(s^*(0))$ from (A5),

$$\Delta > [C_1^* - c(s^*(0),0)][Q^*(1)(s^*(1)/s^*(0))-Q^*(0)] + [D(E^*(0))-D(E^*(1))] \tag{A10}$$
$$= [D(E^*(0))-t_e(s^*(0))E^*(0)] - [D(E^*(1))-t_e(s^*(0))E^*(1)] = \int_{E^*(1)}^{E^*(0)} [D'(E)-t_e(s^*(0))]dE$$

Eq. (A10), Result VIII, and $E^*(1) \leq E^*(0)$ (Assumption 1) now imply

$$\Delta > \int_{E^*(1)}^{E^*(0)} [D'(E)-t_1]dE \geq 0, \tag{A11}$$

where the last inequality is due to the definition of t_1 $(=D'(E^*(1)))$, $D''() \geq 0$, and $E^*(0) \geq E^*(1)$ (Assumption 1). (A11) establishes the desired inequality, (A6). *QED*.

B. Section VI: Proof that Firm 1 Will Truthfully Report Under Optimal Government Policies

Firm 1's profit from a report of δ_{1r}, given a technology δ_1, are (with subsequent truthful reporting by firm 2):

$$\pi^*(\delta_{1r};\delta_1) \equiv E_{I_2} \{ \int_0^{\min(\delta_{1r},\delta_1)} \pi(\delta_{1r},\delta_2;\delta_1)\, f(\delta_2;I_2)\, d\delta_2 \},$$

where E_{I_2} is firm 1's expectation operator over firm 2's R&D investment I_2 and, per the logic given in the paper, $\pi(\delta_{1r},\delta_2;\delta_1)=0$ if $\delta_{1r}<\delta_2$ or $\delta_1\leq\delta_2$. For $\delta_{1r}<\delta_1$,

$$\partial\pi^*()/\partial\delta_{1r} = E_{I_2} \{ \pi(\delta_{1r},\delta_{1r};\delta_1)f(\delta_{1r};I_2) + \int_0^{\delta_{1r}} [\partial\pi(\delta_{1r},\delta_2;\delta_1)/\partial\delta_{1r}]\, f(\delta_2;I_2)\, d\delta_2 \} > 0,$$

where the inequality is due to $\pi(\delta_{1r},\delta_2;\delta_1)>0$ for $\delta_2=\delta_{1r}<\delta_1$, and the analog for eq. (28) ($\partial\pi(\delta_{1r},\delta_2;\delta_1)/\partial\delta_{1r}>0$ for $\delta_2\leq\delta_{1r}<\delta_1$). For $\delta_{1r}\geq\delta_1$,

$$\partial\pi^*()/\partial\delta_{1r} = E_{I_2} \{ \int_0^{\delta_1} [\partial\pi(\delta_{1r},\delta_2;\delta_1)/\partial\delta_{1r}]\, f(\delta_2;I_2)\, d\delta_2 \} \leqq 0 \quad \text{when} \quad \delta_{1r} \geqq \delta_1,$$

with the sign relation again due to the analog for eq. (28) ($\partial\pi(\delta_{1r},\delta_2;\delta_1)/\partial\delta_{1r} \leqq 0$ when $\delta_{1r} \geqq \delta_1>\delta_2$). Thus, the expected firm 1 profit $\pi^*()$ is maximized with a truthful report, $\delta_{1r}=\delta_1$.

C. Extension: Efficient Taxes and Standards Without Assumption 1

i. Section V. Proposition 1, Observations 1-3, and Corollary 1 give us the following revised statement of Proposition 2:

Proposition 2'. If $E^*(1)<E^*(0)$ (or $\pi_B(s^*(0))<W^*(1)-W^*(0)$), then fully efficient outcomes are produced by the following policy of emission taxes and per-unit-output emission standards: (1) Pigovian emission taxes (with optional first-best emission standards) in the symmetric technology cases (A) and (C); and (2) for the asymmetric technology case (B), a first-best "winner" standard, a more lax environmental standard for the "loser," and an emission tax that is less than its Pigovian counterpart: $s_W=s^*(1)$, $s_L=s_L^* \in(s^*(1),s^*(0))$, $t_e=t_e(s_L^*)<t_1$.

When the prior conditions of Proposition 2' are violated, we impose a plausible restriction on post-innovation emission standards in our asymmetric technology case (B): Because it is implausible for innovation to spur a relaxation in environmental performance standards, we restrict the case (B) emission standards to be no more lax than would prevail without innovation, $\max(s_W,s_L)\leq s^*(0)$.

In view of Proposition 2', the remaining possibility is that emissions rise with innovation ($E^*(1)>E^*(0)$) and, in addition, the most lax loser standard possible ($s_L=s^*(0)$) still provides firms with an incentive to overinvest in R&D ($\pi_B(s^*(0))>W^*(1)-W^*(0)$). For this circumstance, the following policy can optimally counter the persistent overinvestment problem: (1) set the environmental standards to maximally differentiate between the winning and losing firms, $s_W=s^*(1)$ and $s_L=s^*(0)$; (2) lower the emission tax t_e below its ex-post efficient level, $t_e<t_e(s^*(0))$; and (3) compensate for the lower emission tax by levying a positive output tax, $t_q>0$, that preserves efficient pricing,

(C1) $$t_q=t_q(t_e)\ t_q:\ c(s^*(0),0)+t_e s^*(0)+t_q = C_1^* .$$

Because the losing firm emits more pollutants per-unit-output than does the winning firm ($s^*(0)>s^*(1)$), the reduced emission tax lowers the losing firm's per-unit-output costs more than it does for the winner. The winner's reduced cost advantage in turn lowers its profits and the associated incentive to invest in R&D. By lowering the emission tax sufficiently far (and raising the output tax in tandem), the overinvestment problem can be cured. Formally, this policy gives rise to winner profits of

(C2) $\pi_B^{so}(t_e) = \{[c(s^*(0),0)+t_e s^*(0)+t_q(t_e)] - [c(s^*(1),1)+t_e s^*(1)+t_q(t_e)]\ Q^*(1)$

$= \{c(s^*(0),0)-c(s^*(1),1) + t_e(s^*(0)-s^*(1))\}Q^*(1),$

where the second equality is obtained by substituting for $t_q()$ from eq. (C1), and differentiating reveals that the winner's profits decline with a reduced emission tax, $\partial\pi_B^{so}(t_e)\ /\partial t_e=(s^*(0)-s^*(1))Q^*(1)>0$.

Proposition 2". If $E^*(1)>E^*(0)$ and $\pi_B(s^*(0))>W^*(1)-W^*(0)$, then fully efficient outcomes can be prompted by a policy of the following form: (1) Pigovian emission taxes in cases (A) and (C); and (2) for case (B), a first-best "winner" standard ($s_W=s^*(1)$), no change in the "loser" standard ($s_L=s^*(0)>s^*(1)$), a low emission tax ($t_e<t_e(s^*(0))<\max(t_0,t_1)$), a positive output tax ($t_q=t_q(t_e)>0$), and a combined per-unit-output tax (for the winner) that is less than the marginal pollution damage ($t_q+t_e s^*(1)< D'(E^*(1))s^*(1)$).

ii. Section VI. All in Section VI extends directly, with one change: The optimal Section V policy must allow for output taxes (as described in Proposition 2" above). Specifically, our Section V policy of taxes and standards now stipulates the emission tax t_e, output tax t_q, "winner" standard s_W, and "loser" standard s_L that satisfy: (i) efficient "winner" emissions, $s_W=s^*(\delta_{Wr})$, (ii) efficient pricing (with $s_L>s_W$, $t_e<t_{\delta_{Wr}}$, and $t_q\geq 0$)

(C3) $c(s_L,\delta_{Lr}) + t_e s_L + t_q = C^*_{\delta_{Wr}}$,

and (iii) a "winner" payoff exactly equal to the societal gains from the excess innovation, $\delta_{Wr}-\delta_{Lr}$:

(C4) Winner Payoff $= Q^*(\delta_{Wr})\ \{C^*_{\delta_{Wr}} -c(s^*(\delta_{Wr}),\delta_{Wr})-t_e s^*(\delta_{Wr})-t_q\} = W^*(\delta_{Wr})-W^*(\delta_{Lr})$

For notational convenience, we will denote these (generalized) Section V policies by $\{Z(\delta_{Wr},\delta_{Lr})\}\equiv\{s_W=s^*(\delta_{Wr}),s_L(\delta_{Wr},\delta_{Lr}),t_e(\delta_{Wr},\delta_{Lr}),t_q(\delta_{Wr},\delta_{Lr})\}$. (As above, we will uniquely identify these policies with the restriction that $s_L\leq s^*(0)$ and appealing to positive output taxes only when they are needed to equate the rents of successful innovators with the societal gains from the innovation.) With this revised {Z()}, we have:

Proposition 3'. Given the optimal environmental policies, $\{Z(\delta_{Wr},\delta_{Lr})\}$, and the technology verification requirement described in the paper, there is a subgame perfect equilibrium in which firms truthfully reveal their technologies to the government and, hence, first-best outcomes are attained.

iii. Proofs of Results Without Assumption 1.

Proof of Proposition 1 in the paper (No Assumption 1). Define

(C5) $$X_1 \equiv (C_0^* - C_1^*)Q^*(1) \quad , \quad X_2 \equiv (C_0^* - [c(s^*(1),0)+t_1 s^*(1)])\,Q^*(1),$$

$$X_3 \equiv \int_{Q(C_0^*)}^{Q(C_1^*)} (P(Q)-C_0^*)\,dQ < 0 \quad ,$$

$$X_4 \equiv \{[D'(E^*(1))E^*(1)-D(E^*(1))]-[D'(E^*(0))E^*(0)-D(E^*(0))]\},$$

where $X_3<0$ is due to $C_0^* > C_1^*$ (and hence, $P(Q)<C_0^*$ for $Q\in(Q(C_0^*),Q(C_1^*))$). Noting that $W^*(\delta)$ can be written as

$$W^*(\delta) = \int_0^{Q(C_\delta^*)} P(z)dz - C_\delta^*\,Q(C_\delta^*) + \{D'(E^*(\delta))E^*(\delta)-D(E^*(\delta))\},$$

we can expand $W^*(1)-W^*(0)$ as follows:

(C6) $$W^*(1)-W^*(0) = X_1+X_3+X_4$$

Similarly, expanding $\pi_B(s^*(1))$ in (22),

$$\pi_B(s^*(1)) = X_1-X_2$$

With $X_3<0$, and $X_4\leq(t_1-t_0)E^*(1)$, the following is a sufficient condition for overinvestment to occur:

(C7) $$X_2+(t_1-t_0)E^*(1) < 0 \Rightarrow X_2+X_3+X_4 < 0 \Rightarrow \pi_B(s^*(1)) > W^*(1)-W^*(0)$$

Expanding the left-hand-side of (C7) (substituting for $C_0^* = c(s^*(0),0)+t_0 s^*(0)$),

(C8) $$X_2+(t_1-t_0)E^*(1) = Q^*(1)\{[c(s^*(0),0)-c(s^*(1),0)]+t_0[s^*(0)-s^*(1)]\}$$

$$= Q^*(1) \int_{s^*(1)}^{s^*(0)} \{c_s(s,0)+t_0\} ds < 0,$$

where the inequality follows from $c_s(s^*(0),0)+t_0=0$ (by the definition of $s^*(0)=s(t_0,0)=\text{argmin } c(s,0)+t_0 s$), $c_{ss}>0$, and $s^*(0)>s^*(1)$. *QED*.

Note: Proofs of Observation 1, Observation 2, and Corollary 1 do not rely upon Assumption 1.

Proof of Proposition 2". At $t_e^0 \equiv t_e(s^*(0))$ (as defined in eq. (21)), we have (by construction and assumption) $t_q(t_e^0)=0$ and (with $\pi_B^{so}(t_e)$ as defined in (C2) above and $\pi_B(s_L)$ as defined in eq. (22))

$$\pi_B^{so}(t_e^0) = \pi_B(s^*(0)) > W^*(1)-W^*(0).$$

Furthermore, at $t_e^1 \equiv -[c(s^*(0),0)-c(s^*(1),1)]/[s^*(0)-s^*(1)] < t_e^o$, we have

$$\pi_B^{so}(t_e^1) = 0 < W^*(1)-W^*(0).$$

Therefore, by the Intermediate Value Theorem, there is a $t_e^* \in (t_e^1, t_e^0)$ such that

(C9) $$\pi_B^{so}(t_e^*) = W^*(1)-W^*(0).$$

By (C9) and eq. (C1), the following policy yields a first-best: $t_e=t_e^*$, $t_q=t_q(t_e^*)$, $s_L=s^*(0)$, and $s_W=s^*(1)$. With $t_e^* < t_e^0$ and $dt_q(t_e)/dt_e<0$ (by eq. (C1)), we have $t_q(t_e^*)>0$. Finally, by eq. (C9), the definition of $\pi_B^{so}(t_e)$ in (C2), and $W^*(1)-W^*(0)>0$, we have

(C10) $$\pi_B^{so}(t_e^*) > 0 \Rightarrow c(s^*(0),0)+t_e^* s^*(0) > c(s^*(1),1)+t_e^* s^*(1).$$

Furthermore, eq. (C10) and eq. (C1) imply

$$t_q(t_e^*) = C_1^* - [c(s^*(0),0)+t_e^* s^*(0)] < C_1^* - [c(s^*(1),1)+t_e^* s^*(1)] = t_1 s^*(1)-t_e^* s^*(1),$$

which gives us the final inequality in Proposition 2" ($t_q+t_e s^*(1)<t_1 s^*(1)$). *QED*.