# Web Appendix for:

## Entry and Competition in Local Hospital Markets


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**Abstract**

In this appendix, we derive the likelihood function for the estimation in our paper in the *Journal of Industrial Economics*, titled above.

# Derivation of the likelihood function

The two equations we wish to estimate are equation 1, an ordered-probit entry equation, and equation 2, a linked demand equation which has both selection bias and endogeneity of the market structure dummies.

$$\begin{aligned} & Y\lambda + X\left(\delta_X + \alpha_X\right) + W\left(\delta_W + \alpha_W - \gamma_W\right) \\ & + \delta_N - \alpha_N - \gamma_N - \ln N + \epsilon_S + \epsilon_d + \epsilon_V - \epsilon_F > 0 \end{aligned} \tag{1}$$

$$\ln Q_N = Y\lambda + X\delta_X + W\delta_W + \delta_N + \epsilon_Q \tag{2}$$

Because $\nu_Q$, $\nu_\Pi$, and $\eta$ are mutually independent, $\epsilon_\Pi$ and $\epsilon_Q$ are independent once we condition on $\eta$. Consider now the contribution (conditional on $\eta$) to the likelihood function of a market with $N = 0$:

$$\begin{aligned} P\left\{N = 0|\eta\right\} =& P\left\{Y\lambda + X\mu_X + W\mu_W + \epsilon_\Pi < \mu_1|\eta\right\} \\ P\left\{N = 0|\eta\right\} =& P\left\{Y\lambda + X\mu_X + W\mu_W + \nu_\Pi + r\eta < \mu_1|\eta\right\} \\ P\left\{N = 0|\eta\right\} =& \Phi\left(\mu_1 - Y\lambda - X\mu_X - W\mu_W - r\eta\right) \end{aligned}$$

The contribution (conditional on $\eta$) to the likelihood function of a market with $N = n$ is:

$$P\left\{N = n|\eta\right\} f\left(\ln Q|\eta\right) = P\left\{\mu_n < Y\lambda + X\mu_X + W\mu_W + \nu_\Pi + r\eta < \mu_{n+1}|\eta\right\} f\left(\ln Q|\eta\right)$$

$$= \begin{pmatrix} \Phi\left(\mu_{n+1} - Y\lambda - X\mu_X - W\mu_W - r\eta\right) \\ -\,\Phi\left(\mu_n - Y\lambda - X\mu_X - W\mu_W - r\eta\right) \end{pmatrix} \frac{1}{\sigma_{\nu_Q}} \phi\left(\frac{\ln Q - Y\lambda - X\delta_X - W\delta_W - \delta_N - \eta}{\sigma_{\nu_Q}}\right)$$

Finally, the contribution (conditional on $\eta$) to the likelihood function of a market with $N = \overline{n}$, where $\overline{n}$ is the "top" category in the ordered probit, is:

$$P\left\{N = \overline{n}|\eta\right\} f\left(\ln Q|\eta\right) = P\left\{\mu_{\overline{n}} < Y\lambda + X\mu_X + W\mu_W + \nu_\Pi + r\eta|\eta\right\} f\left(\ln Q|\eta\right)$$

$$= \left(1 - \Phi\left(\mu_{\overline{n}} - Y\lambda - X\mu_X - W\mu_W - r\eta\right)\right) \frac{1}{\sigma_{\nu_Q}} \phi\left(\frac{\ln Q - Y\lambda - X\delta_X - W\delta_W - \delta_N - \eta}{\sigma_{\nu_Q}}\right)$$

Now let us turn to $\eta$. Let $\eta$ be distributed with a distribution function $F(\eta;\beta)$ which depends on parameters $\beta$. Then the contribution of an observation with $N = n$ where $n$ is neither zero nor the top category would be:

$$\int_\eta P\left\{N = n|\eta\right\} f\left(\ln Q|\eta\right) dF(\eta;\beta) =$$

$$\int_\eta \begin{pmatrix} \Phi\left(\mu_{n+1} - Y\lambda - X\mu_X - W\mu_W - r\eta\right) \\ -\,\Phi\left(\mu_n - Y\lambda - X\mu_X - W\mu_W - r\eta\right) \end{pmatrix} \frac{1}{\sigma_{\nu_Q}} \phi\left(\frac{\ln Q - Y\lambda - X\delta_X - W\delta_W - \delta_N - \eta}{\sigma_{\nu_Q}}\right) dF(\eta;\beta)$$

To arrive at the unconditional contribution to the likelihood function, we must integrate over $\eta$. Rather than assuming a particular functional form for the distribution of $\eta$, we choose to approximate this distribution using a discrete factor approximation (Heckman and Singer, 1984; Mroz and Guilkey, 1992). This entails using a multinomial distribution for $\eta$ with $K$ points of support:

$$\eta = \begin{cases} \beta_1 & \text{with probability} \quad p_1 \\ \beta_2 & \text{with probability} \quad p_2 \\ \beta_3 & \text{with probability} \quad p_3 \\ \vdots & \qquad \vdots \qquad\qquad \vdots \\ \beta_K & \text{with probability} \quad p_K \end{cases}$$

The use of this distribution with $K$ points of support introduces $2K$ additional parameters, the $K$ $\beta$s and the $K$ $p$s. Accounting for the fact that probabilities must sum to one and the mean of $\eta$ must be zero, there are $2(K-1)$ additional parameters.

At last, the unconditional contribution to the likelihood function of an observation with $N = n$ where $n$ is neither zero nor the top category is:

$$\int P\{N = n|\eta\} f(\ln Q|\eta)\, dF(\eta;\beta) =$$

$$\sum_{k=1}^{K} p_k \begin{pmatrix} \Phi(\mu_{n+1} - Y\lambda - X\mu_X - W\mu_W - r\beta_k) \\ -\Phi(\mu_n - Y\lambda - X\mu_X - W\mu_W - r\beta_k) \end{pmatrix} \frac{1}{\sigma_{\nu_Q}} \phi\left(\frac{\ln Q - Y\lambda - X\delta_X - W\delta_W - \delta_N - \beta_k}{\sigma_{\nu_Q}}\right)$$

The contributions for observations with $N = 0$ and with $N$ equal to the top category in the ordered probit may be derived similarly. It is this likelihood function which we take to the data. It remains to choose a $K$, and we follow the prior literature in that we increase $K$ until the likelihood function no longer rises appreciably with further increases in $K$. In our case, raising $K$ from six to seven resulted in the likelihood function rising by approximately 0.05, so we set $K$ equal to seven.

# References

Heckman, J. and Singer, B. (1984). A method for minimizing the impact of distributional assumptions in econometric models for duration data. *Econometrica*, 52:271–320.

Mroz, T. and Guilkey, D. (1992). Discrete factor approximations for use in simultaneous equation models with both continuous and discrete endogenous variables. unpublished paper, Department of Economics, University of North Carolina, Chapel Hill.