Supplemental Materials for Anette Boom,"On the Desirability of Compatibility with Product Selection", *The Journal of Industrial Economics* Volume 69, March 2001, pp. 85-96.

Incompatible Components

Given $x_i \leq x_j$ and $y_i \leq y_j$, the most preferred variants $(\tilde{x}, \tilde{y})$ of the consumers who are indifferent between buying from firm i or from firm j can be determined from (1) and are given by $(x, \hat{y}(x, p_i, p_j, x_i, x_j, y_i, y_j))$ with:

$$(1) \quad \hat{y}(x, p_i, p_j, x_i, x_j, y_i, y_j) = \frac{p_j - p_i - 2x(x_j - x_i) + x_j^2 - x_i^2 + y_j^2 - y_i^2}{2(y_j - y_i)}$$

The function $\hat{y}(\cdot)$ is linearly decreasing in x. Every consumer whose most preferred product specification $(\tilde{x}, \tilde{y})$ lies to the south-west of $\hat{y}(\cdot)$ buys the whole system from firm i, whereas those whose most preferred product variant $(\tilde{x}, \tilde{y})$ is located to the north-east buy from firm j.

For the equilibrium positions $(x_i, y_j) = (1/2, 0)$ and $(x_j, y_j) = (1/2, 1)$ we obtain:

$$(2) \qquad \hat{y}(x, p_i, p_j, 1/2, 1/2, 0, 1) = \frac{p_j - p_i + 1}{2}$$

Note that (2) describes a horizontal line in the product space. Thus, the profit function of the two firms for $(x_i, y_i) = (1/2, 0)$ and $(x_j, y_j) = (1/2, 1)$, are given by:

$$(3) \qquad \pi_i(p_i, p_j, 1/2, 1/2, 0, 1) = \frac{p_i(p_j - p_i + 1)}{2}$$

$$(4) \qquad \pi_j(p_i, p_j, 1/2, 1/2, 0, 1) = \frac{p_j(p_i - p_j + 1)}{2}$$

Solving the first order conditions of profit maximisation with respect to prices results in (2). Substituting (2) into (3) and (4) yields (3). The same results would hold true, if we would have considered the other possible equilibrium locations of Proposition 1. Taking into account the surplus function (1) and (2) at the equilibrium prices (2), the aggregate consumer's surplus (CS) is represented by:

$$CS^* = \bar{v} - 1 - \int_0^1 \int_0^{\frac{1}{2}} (-\tilde{x})^2 + (\frac{1}{2} - \tilde{y})^2 d\tilde{x} d\tilde{y} - \int_0^1 \int_{\frac{1}{2}}^1 (1 - \tilde{x})^2 + (\frac{1}{2} - \tilde{y})^2 d\tilde{x} d\tilde{y},$$

which yields (4).

Compatible Components

Given $x_i \leq x_j$ and $y_i \leq y_j$, the position $(\tilde{x}, \tilde{y})$ of the consumers who are indifferent about buying component x from firm i or firm j is, from (1), given by $(\check{x}(q_i, q_j, x_i, x_j), \tilde{y})$ with:

$$\check{x}(q_i, q_j, x_i, x_j) = \frac{q_j - q_i + x_j^2 - x_i^2}{2(x_j - x_i)} \text{ and } \tilde{y} \in [0, 1]. \tag{5}$$

The position $(\tilde{x}, \tilde{y})$ of the consumers who are indifferent between buying component y from either firm is given by $(\tilde{x}, \check{y}(r_i, r_j, y_i, y_j))$ with:

$$\check{y}(r_i, r_j, y_i, y_j) = \frac{r_j - r_i + y_j^2 - y_i^2}{2(y_j - y_i)} \text{ and } \tilde{x} \in [0, 1]. \tag{6}$$

In (5) and (6) q_i is the price of component x and r_i the one of component y of firm i, $i \in \{1, 2\}$. For the profit functions of the two firms we obtain:

$$\pi_i(q_i, q_j, r_i, r_j, x_i, x_j, y_i, y_j) = q_i \check{x}(q_i, q_j, x_i, x_j) + r_i \check{y}(r_i, r_j, y_i, y_j)$$

$$\pi_j(q_i, q_j, r_i, r_j, x_i, x_j, y_i, y_j) = q_j \left[1 - \check{x}(q_i, q_j, x_i, x_j)\right] + r_j \left[1 - \check{y}(r_i, r_j, y_i, y_j)\right]$$

From the first order conditions of profit maximisation with respect to prices we derive a unique Bertrand-Nash equilibrium for any possible location of the two firms with:[1]

$$q_i = \frac{2(x_j - x_i) + x_j^2 - x_i^2}{3} \qquad r_i = \frac{2(y_j - y_i) + y_j^2 - y_i^2}{3} \tag{7}$$

$$q_j = \frac{4(x_j - x_i) - x_j^2 + x_i^2}{3} \qquad r_j = \frac{4(y_j - y_i) - y_j^2 + y_i^2}{3} \tag{8}$$

Substituting (7) and (8) into the profit functions yields:

$$\pi_i(x_i, x_j, y_i, y_j) = \frac{(x_j - x_i)(2 + x_i + x_j)^2}{18} + \frac{(y_j - y_i)(2 + y_i + y_j)^2}{18}$$

$$\pi_j(x_i, x_j, y_i, y_j) = \frac{(x_j - x_i)(x_i + x_j - 4)^2}{18} + \frac{(y_j - y_i)(y_i + y_j - 4)^2}{18}$$

The profit $\pi_i(x_i, x_j, y_i, y_j)$ is monotonously decreasing for all x_i, y_i with $0 < x_i < x_j < 1$ and $0 < y_i < y_j < 1$, and $\pi_j(x_i, x_j, y_i, y_j)$ is monotonously increasing for all x_j, y_j in the relevant range $0 < x_i < x_j < 1$ and $0 < y_i < y_j < 1$ with $x_i \in [0, 1]$ and $y_i \in [0, 1]$. Since the analogous analysis for $x_i < x_j$ and $y_i > y_j$ yields the same result, firms choose the positions, given in Proposition 2. Substituting these positions into the profit functions and into the functions (7) and (8) yields (7) and (6). Substituting equilibrium

[1]Existence of a Bertrand-Nash equilibrium in pure strategies is guaranteed because both profit functions are quasi-concave and continuous in prices.

locations and prices yields $\check{x} = \frac{1}{2}$ and $\check{y} = \frac{1}{2}$. Thus, the aggregate consumer's surplus is given by:

$$
\begin{aligned}
CS^{**} \;&=\; \bar{v} - 2 - \int_0^{\frac{1}{2}} \int_0^{\frac{1}{2}} (-\tilde{x})^2 + (-\tilde{y})^2 d\tilde{x} d\tilde{y} - \int_0^{\frac{1}{2}} \int_{\frac{1}{2}}^1 (1-\tilde{x})^2 + (-\tilde{y})^2 d\tilde{x} d\tilde{y} \\
&\quad - \int_{\frac{1}{2}}^1 \int_{\frac{1}{2}}^1 (1-\tilde{x})^2 + (1-\tilde{y})^2 d\tilde{x} d\tilde{y} - \int_{\frac{1}{2}}^1 \int_0^{\frac{1}{2}} (\tilde{x})^2 + (1-\tilde{y})^2 d\tilde{x} d\tilde{y},
\end{aligned}
$$

which is equivalent to (8).

Different Timing Assumptions

Simultaneous Decision on Compatibility and the Product Design: If incompatibility is an equilibrium, firms choose the same positions as in Proposition 1, and, if compatibility is an equilibrium, it is fully characterised by Proposition 2. In the latter the enforcing firm should not have an incentive to refrain from doing so and to choose the best locational response to the rival's position in one of the corners. The best response would be the midpoint of one side which is farthest away from that corner (see Lemma 2 in Tabuchi [1994]). The deviating firm's profit would be 169/288. Thus, equilibrium with compatibility exists as long as $1-C > 169/288 \Leftrightarrow C < 119/288$. Incompatibility is an equilibrium if no firm has an incentive to initiate compatibility and to move its product to the best response to its rival's position on the midpoint of one side. This would be one of the corners farthest away from the rival's position (see the previous section in the Appendix). The deviating firm's profit would be $97/144 - C$. This exceeds the profit of 1/2 without compatibility as long as $C < 25/144$. Thus, for $C < 25/144$ only the equilibria with compatibility, for $25/144 \leq C \leq 119/288$ both types of equilibria with and without compatibility, and for $C > 119/288$ only the equilibria with incompatibility exist.

Selection of the Product Design before the Decision on Compatibility: Here each firm could, for sufficiently low compatibility costs, trigger the preferred type of equilibrium by its choice of location in the product space. If it moves to one of the corners of the product space in the first stage of the game, this would induce the rival to locate at the diagonal corner as long as $C < 119/288$ (see the argument above), even if he holds the most pessimistic expectation that he will have to initiate compatibility later on. Given these locations of products, one of the firms will, indeed, enforce compatibility in the second stage of the game. Even if this is the deviating firm it will gain compared to the equilibrium without compatibility. It is not possible to derive any results for $C > 119/288$ without further assumptions on the firms' expectation which firm would have to bear the compatibility costs later on.