Supplemental material for Marc Bourreau, Frago Kourandi and Tommaso Valletti, "Net Neutrality with Competing Internet Platform," *The Journal of Industrial Economics*.

I. SEQUENTIAL GAME

In this annex, we show that our main findings hold when ISPs choose capacity investments before setting their prices to end users and CPs. We first solve for the equilibrium of the sequential game under the net neutrality regime, and then under the discriminatory regime. Finally, we compare equilibrium investments in the two regimes.

I(i). *Net neutrality*

In the net neutrality regime, there is a single lane for Internet traffic, and CPs pay no fee to the ISPs. We study the following three-stage game:

1. The two ISPs choose their capacities, μ_A^N and μ_B^N.
2. The two ISPs set the subscription fees to the end users, p_A^N and p_B^N.
3. The CPs choose which ISP(s) to connect to (if any), and the end users choose which ISP to subscribe to.

We proceed backwards to solve for the symmetric subgame perfect equilibrium.[1]

Stage 3: Content providers' and end users' decisions. This stage is the same as Stage 2 of our baseline model. By solving for the cubic equation (10) of the paper, we can obtain an explicit expression for the indifferent consumer, $\widetilde{x}^N = \widetilde{x}^N \left(p_A^N, p_B^N, \mu_A^N, \mu_B^N\right)$. The number of end users of ISP A and ISP B are then $x_A^N = \widetilde{x}^N$ and $x_B^N = 1 - \widetilde{x}^N$, respectively. Note that $\widetilde{x}^N$ is equal to $1/2$ under symmetry, i.e., when $p_A^N = p_B^N$ and $\mu_A^N = \mu_B^N$.

Stage 2: Subscription fees. At Stage 2, the ISPs set the subscription fees to the end users. The maximization problem of ISP $i = A, B$ is as follows

$$\max_{p_i^N} \Pi_i^N = p_i^N x_i^N - C\left(\mu_i^N\right),$$

[1] When we consider (out of equilibrium) asymmetric situations in the multi-stage games, expressions and computations become complex. For this reason, we describe here the procedure we followed and the main findings, but do not provide the full expressions. Of course we can provide them upon request in case there is interest in fuller details.

where $x_i^N = x_i^N \left(p_A^N, p_B^N, \mu_A^N, \mu_B^N\right)$ from Stage 3. From the first-order conditions,

$$\frac{\partial \Pi_i^N}{\partial p_i^N} = x_i^N + p_i^N \frac{\partial x_i^N}{\partial p_i^N} = 0,$$

we obtain *implicitly* (i.e., using the implicit function theorem) an expression for the subscription fees $p_i^N \left(\mu_A^N, \mu_B^N\right)$.

Stage 1: Investment in capacity. At Stage 1, the two ISPs choose their investments in capacity. The maximization problem of ISP i can be expressed as follows

$$\max_{\mu_i^N} \Pi_i^N = p_i^N x_i^N - C\left(\mu_i^N\right),$$

where $x_i^N = x_i^N \left(p_A^N, p_B^N, \mu_A^N, \mu_B^N\right)$ is obtained explicitly from the third stage and $p_i^N \left(\mu_A^N, \mu_B^N\right)$ is obtained implicitly from the second stage. Applying the envelope theorem, the first-order condition with respect to the investment in capacity is

$$\frac{\partial \Pi_i^N}{\partial \mu_i^N} = p_i^N \left(\frac{\partial x_i^N}{\partial \mu_i^N} + \frac{\partial x_i^N}{\partial p_j^N}\frac{\partial p_j^N}{\partial \mu_i^N}\right) - C'\left(\mu_i^N\right) = 0.$$

The derivatives $\partial x_i^N / \partial p_j^N$ and $\partial x_i^N / \partial \mu_i^N$ are obtained by direct differentiation of x_i^N (from Stage 3) with respect to p_j^N and μ_i^N, respectively. Since we do not have an explicit solution for the subscription fees $p_i^N \left(\mu_A^N, \mu_B^N\right)$, we use the implicit function theorem and apply Cramer's rule to the system of first-order conditions with respect to subscription fees (Stage 2) in order to determine the derivative $\partial p_j^N / \partial \mu_i^N$. Under symmetry, we find that

$$\frac{\partial p_j^N}{\partial \mu_i^N} = \frac{4(d+v)(\lambda-1)}{3(2+\lambda)^2}.$$

We can then replace for the expressions of the derivatives in the first-order condition with respect to the investment in capacity (Stage 1), and by imposing symmetry, we obtain the symmetric equilibrium level of investment in capacity in the sequential game,

$$\mu_S^N = (C')^{-1}\left(\frac{(d+v)(4+5\lambda)}{3(2+\lambda)^2}\right).$$

I(ii). *Discrimination*

In the discriminatory regime, each ISP offers a priority lane and a non-priority lane to CPs. The CPs that opt for priority at ISP i pay a fixed fee f_i, whereas the non-priority lane is offered for free. We modify our three-stage game accordingly:

1. The two ISPs choose their capacities, μ_A^D and μ_B^D.
2. The two ISPs set their subscription fees to the end users, p_A^D and p_B^D, as well as the fees for their priority lanes, f_A and f_B.
3. The CPs choose which ISP(s) to connect to (if any) and whether to pay for priority, and the end users choose which ISP to subscribe to.

Stage 3: Content providers' and end users' decisions. This stage is the same as the second stage of our baseline model. By solving for the cubic equation (19) of the paper, we obtain an explicit expression for the indifferent consumer, $\widetilde{x}^D = \widetilde{x}^D\left(p_A^N, p_B^N, \mu_A^N, \mu_B^N\right)$, which is independent of the priority fees. The number of users of ISP A and ISP B are then $x_A^D = \widetilde{x}^D$ and $x_B^D = 1 - \widetilde{x}^D$.

Stage 2: ISPs' pricing decisions. At Stage 2, the two ISPs choose simultaneously their subscription and priority fees. The maximization problem of ISP i can be expressed as follows:

$$\max_{p_i^D,\, f_i} \Pi_i^D = p_i^D x_i^D + \left(\overline{h}_i^D - \widetilde{h}_i\right) f_i - C\left(\mu_i^D\right),$$

where $x_i^D = x_i^D(p_A^D, p_B^D, \mu_A^D, \mu_B^D)$ from the third stage. The first-order conditions are

$$\begin{aligned} \frac{\partial \Pi_i^D}{\partial p_i^D} &= x_i^D + \left(p_i^D + \frac{\partial((\overline{h}_i^D - \widetilde{h}_i) f_i)}{\partial x_i^D}\right) \frac{\partial x_i^D}{\partial p_i^D} = 0, \\ \frac{\partial \Pi_i^D}{\partial f_i} &= \frac{\partial((\overline{h}_i^D - \widetilde{h}_i) f_i)}{\partial f_i} = 0. \end{aligned}$$

Since x_i^D is independent of the priority fees, by solving for $\partial \Pi_i^D / \partial f_i = 0$ we can express the equilibrium priority fee as a function of x_i^D:

$$f_i = \frac{a x_i^D \lambda (1 + x_i^D \lambda - \sqrt{1 + x_i^D \lambda})}{1 + x_i^D \lambda}.$$

Replacing for the optimal f_i's into the first-order conditions $\partial \Pi_i^D / \partial p_i^D = 0$, we obtain a system of two equations with two unknowns that gives *implicitly* the subscription fees $p_i^D\left(\mu_A^D, \mu_B^D\right)$.

Stage 1: Investment in capacity. At Stage 1, the two ISPs decide on their investments in capacity. The maximization problem of ISP i is

$$\max_{\mu_i^D} \Pi_i^D = p_i^D x_i^D + \left(\overline{h}_i^D - \widetilde{h}_i\right) f_i - C\left(\mu_i^D\right),$$

where $x_i^D = x_i^D(p_A^D, p_B^D, \mu_A^D, \mu_B^D)$ is obtained explicitly from the third stage and $p_i^D\left(\mu_A^D, \mu_B^D\right)$ is obtained implicitly from the second stage. Applying the envelope theorem, the first-order condition with respect to the investment in capacity, $\partial \Pi_i^D / \partial \mu_i^D = 0$, becomes

$$p_i^D \left(\frac{\partial x_i^D}{\partial \mu_i^D} + \frac{\partial x_i^D}{\partial p_j^D}\frac{\partial p_j^D}{\partial \mu_i^D}\right) + f_i \frac{\partial(\overline{h}_i^D - \widetilde{h}_i)}{\partial \mu_i^D} + \frac{\partial\left[f_i(\overline{h}_i^D - \widetilde{h}_i)\right]}{\partial x_i^D}\left(\frac{\partial x_i^D}{\partial \mu_i^D} + \frac{\partial x_i^D}{\partial p_j^D}\frac{\partial p_j^D}{\partial \mu_i^D}\right) - C'\left(\mu_i^D\right) = 0.$$

All derivatives are obtained by direct differentiation of the relevant expressions, apart from $\partial p_j^D / \partial \mu_i^D$, as do we do not have an explicit solution for the subscription fees $p_i^D\left(\mu_A^D, \mu_B^D\right)$. We use the implicit function theorem and apply the Cramer's rule to the system of the first-order conditions with respect to the subscription fees (where we have already plugged in the optimal f_i's) in order to determine the derivative $\partial p_j^D / \partial \mu_i^D$. Under symmetry, we obtain

$$\frac{\partial p_j^D}{\partial \mu_i^D} = \frac{N}{\sqrt{t}(2+\lambda)^2\left[3t(2+\lambda)^3 + 2\lambda\mu_i^D\left(6\,(d+v)\,(2+\lambda) + a\lambda\left(3\sqrt{2\,(2+\lambda)} - 8\right)\right)\right]},$$

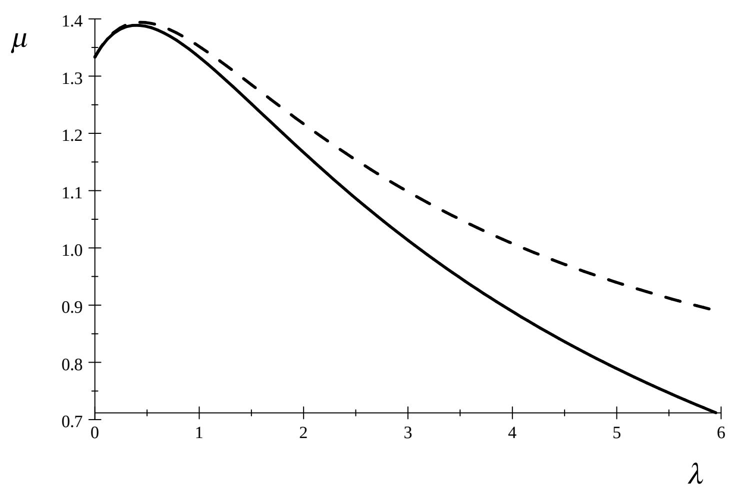
where $N = 4t^{3/2}(2+\lambda)^3((v+d)(\lambda-1)+a\lambda)+4\sqrt{t}\lambda[16a\lambda(v+d)-a^2\lambda^2(14+3\lambda)+4(v+d)^2\left(\lambda^2+\lambda-2\right)]\mu_i^D-2\sqrt{2}a\lambda[8t\sqrt{t(2+\lambda)}+\lambda(12t\sqrt{t(2+\lambda)}+14(v+d)\sqrt{t(2+\lambda)}\mu_i^D+\lambda(6t\sqrt{t(2+\lambda)}+\lambda t\sqrt{t(2+\lambda)}+(v+d-14a)\sqrt{t(2+\lambda)}\mu_i^D))]$. By replacing for the expression of the derivatives in the first-order conditions with respect to the investment in capacity, and by imposing symmetry, we obtain an equation that can be summarized as

$$L\left(\mu_S^D\right)=C'\left(\mu_S^D\right).$$

This expression gives the symmetric equilibrium level of investment in capacity in the sequential move game. We omit the expression of $L\left(\mu_S^D\right)$ here, due to its algebraic complexity.

I(iii). *Net neutrality vs. discrimination*

We can now compare the two alternative regimes and prove that the investment in broadband capacity is higher under the discriminatory regime than under the net neutrality regime in this alternative timing of the game. Since $C''>0$, $(C')^{-1}$ is an increasing function, and therefore, it suffices to prove that the difference $D\equiv L\left(\mu_S^D\right)-(d+v)(4+5\lambda)/\left[3(2+\lambda)^2\right]$ is positive. We find that for $\lambda<14/9$, D is increasing in μ_S^D and that $D\left(\mu_S^D=0\right)\geq 0$. Therefore, D is positive for all $\lambda<14/9$. If $\lambda\geq 14/9$, D is decreasing in μ_i^D and $\lim\limits_{\mu_i^D\to\infty} D\geq 0$. Hence, D is positive for all $\lambda\geq 14/9$ and therefore $D>0$ everywhere. This completes the general proof. We provide below a numerical example for $a=1$, $t=1$, $d=1$, $v=3$ and λ on the horizontal axis. One can observe that $\mu_S^D\geq\mu_S^N$ always holds.



μ_S^N: solid line, μ_S^D: dashed line
$a=1, t=1, d=1, v=3$

II. FIXED ENTRY COST FOR CPs

In this annex, we show that our main findings hold when CPs incur a fixed cost to connect to an ISP. The two-stage games for the net neutrality and discriminatory regimes are the same than in our baseline model. The only difference is that CP h's profits are now given by

$$\Pi_h^N = \begin{cases} a\lambda x_A^N\left(1-hw_A^N\right)+a\lambda x_B^N\left(1-hw_B^N\right)-2K & \text{it connects to both ISPs} \\ a\lambda x_i^N\left(1-hw_i^N\right)-K & \text{if} \quad \text{it connects only to ISP } i \\ 0 & \text{otherwise.} \end{cases}$$

and

$$\Pi_h^D = \begin{cases} a\lambda x_A^D(1-hw_A^P)-f_A+a\lambda x_B^D(1-hw_B^P)-f_B-2K & \text{priority at both ISPs} \\ a\lambda x_i^D(1-hw_i^P)-f_i+a\lambda x_j^D(1-hw_j^{NP})-2K & \text{priority only at ISP } i \\ a\lambda x_A^D(1-hw_A^{NP})+a\lambda x_B^D(1-hw_B^{NP})-2K & \text{if} \quad \text{non-priority at both ISPs} \\ a\lambda x_i^D(1-hw_i^P)-f_i-K & \text{priority at ISP } i\text{, no entry at ISP } j \\ a\lambda x_i^D(1-hw_i^{NP})-K & \text{non-priority at ISP } i\text{, no entry at ISP } j \\ 0 & \text{otherwise,} \end{cases}$$

under net neutrality and discrimination, respectively. The rest of our setting is unchanged. We first solve for the equilibrium under net neutrality, and then under discrimination. Finally, we compare equilibrium investments and entry in the two regimes.

II(i). *Net neutrality*

In this network regime, there is a single lane for Internet traffic, the CPs pay no fee to the ISPs, but CPs now incur a fixed entry cost K when they connect to an ISP. We proceed backwards to solve for the symmetric subgame perfect equilibrium.

Stage 2: Content providers' and end users' decisions. At the second stage, each CP decides whether to multi home, to single home, or to stay out of the market. A CP with congestion sensitivity h connects to ISP i if and only if $a\lambda x_i^N\left(1-hw_i^N\right)-K\geq 0$, that is, iff $h\leq \overline{h}_i^N$, where

$$\overline{h}_i^N = \mu_i^N \frac{a\lambda x_i^N - K}{\lambda x_i^N\left(a+a\lambda x_i^N-K\right)}, \text{ for } i=A,B. \tag{1}$$

Two conflicting effects are at play here: a demand effect and a congestion effect. On the one hand, a higher number of subscribers increases CPs' profits, and hence, entry (*demand effect*). On the other hand, it increases congestion, which reduces entry (*congestion effect*). We find that the congestion effect dominates the demand effect if the fixed entry cost K is sufficiently low ($K\leq \frac{a}{2}(2\lambda x_i+1-\sqrt{4\lambda x_i+1})$) or high ($K\geq \frac{a}{2}(2\lambda x_i+1+\sqrt{4\lambda x_i+1})$); in this case, the number of CPs at ISP i decreases with the number of subscribers on this platform. Otherwise, if the entry cost K takes intermediate values ($K\in(\frac{a}{2}(2\lambda x_i+1-\sqrt{4\lambda x_i+1}),\frac{a}{2}(2\lambda x_i+1+\sqrt{4\lambda x_i+1}))$), the demand effect dominates the congestion effect, and the number of CPs at ISP i increases with the number of subscribers on this platform. Note however that we have also to take into account the constraint that the level of entry by CPs should be positive in the symmetric equilibrium. That is, from (1), it must be that $K\leq a\lambda x_i$. When x_i is around 1/2, this condition becomes $K\leq a\lambda/2$ and we have $\frac{a}{2}(2\lambda x_i+1-\sqrt{4\lambda x_i+1})<a\lambda/2<\frac{a}{2}(2\lambda x_i+1+\sqrt{4\lambda x_i+1})$. Therefore, for $x_i\sim 1/2$, the number of CPs at ISP i decreases with the number of subscribers on this platform for low values of K ($K\leq\frac{a}{2}(2\lambda x_i+1-\sqrt{4\lambda x_i+1})$), and increases with it otherwise. In the rest of the analysis, we assume that $K\leq a\lambda/2$. Note that if $K=0$, the congestion effect is always dominant.

Simultaneously, at Stage 2, each consumer chooses whether to subscribe to ISP A or ISP B. The indifferent consumer $\widetilde{x}^N$ is given by

$$R+v\overline{h}_A^N+\frac{d}{w_A^N}-p_A^N-t\widetilde{x}^N = R+v\overline{h}_B^N+\frac{d}{w_B^N}-p_B^N-t\left(1-\widetilde{x}^N\right). \tag{2}$$

Replacing for $\overline{h}_A^N$ and $\overline{h}_B^N$ into (2), we find that the indifferent consumer is defined implicitly from $F=0$, where

$$
\begin{aligned}
F(\widetilde{x}^N, p_A^N, p_B^N, \mu_A^N, \mu_B^N) \quad \equiv \quad & \frac{\mu_B^N \left(v - d\lambda \left(1-\widetilde{x}^N\right)\right) \left(a\lambda \left(1-\widetilde{x}^N\right) - K\right)}{\lambda \left(1-\widetilde{x}^N\right) \left(a + a\lambda \left(1-\widetilde{x}^N\right) - K\right)} \\
& - \frac{\mu_A^N \left(v - d\lambda \widetilde{x}^N\right) \left(a\lambda \widetilde{x}^N - K\right)}{\lambda \widetilde{x}^N \left(a + a\lambda \widetilde{x}^N - K\right)} + d\left(\mu_B^N - \mu_A^N\right) - t(1-2\widetilde{x}^N) - (p_B^N - p_A^N), \qquad (3)
\end{aligned}
$$

and, therefore, we have $\widetilde{x}^N = \widetilde{x}^N(p_A^N, p_B^N, \mu_A^N, \mu_B^N)$. The number of end users of ISP A and ISP B are then $x_A^N = \widetilde{x}^N$ and $x_B^N = 1-\widetilde{x}^N$, respectively.

Stage 1: ISPs' decisions. At the first stage of the game, the two ISPs compete by choosing an investment in capacity, and by setting a subscription fee to the end users. The maximization problem of ISP i can be expressed as follows

$$\max_{p_i^N,\ \mu_i^N} \Pi_i^N = p_i^N x_i^N - C\left(\mu_i^N\right),$$

where $x_i^N = x_i^N\left(p_A^N, p_B^N, \mu_A^N, \mu_B^N\right)$. The two first-order conditions are

$$\frac{\partial \Pi_i^N}{\partial p_i^N} = x_i^N + p_i^N \frac{\partial x_i^N}{\partial p_i^N} = 0, \qquad (4)$$

and

$$\frac{\partial \Pi_i^N}{\partial \mu_i^N} = p_i^N \frac{\partial x_i^N}{\partial \mu_i^N} - C'\left(\mu_i^N\right) = 0. \qquad (5)$$

Following the same logic as in our baseline model, we obtain the following result.

Proposition 1 Under net neutrality, if the CPs incur a fixed cost K to connect to an ISP, in the symmetric equilibrium, the capacity level, the subscription fee, the number of CPs and the average level of congestion are given by:

$$
\begin{aligned}
\mu_K^N &= (C')^{-1}\left(\frac{a(d+v)\lambda - 2vK}{\lambda(a(2+\lambda)-2K)}\right), \\
p_K^N &= t + \frac{4(-4vK(a-K+a\lambda) + a^2(d+v)\lambda^2)\mu_K^N}{(\lambda(a(2+\lambda)-2K)^2}, \\
\overline{h}_K^N &= \frac{2\left(a\lambda - 2K\right)\mu_K^N}{\lambda(a(2+\lambda)-2K)}, \\
w_K^N &= \frac{a(2+\lambda)-2K}{2a\mu_K^N}.
\end{aligned}
$$

Proof: Since we do not have an explicit solution for market shares x_i^N, we apply the Implicit Function Theorem to equation (3) in order to determine the derivatives $\partial x_i^N/\partial p_i^N$ and $\partial x_i^N/\partial \mu_i^N$, which are then used in the FOCs of Stage 1. We calculate

$$\frac{\partial x_A^N}{\partial p_A^N} = -\frac{\partial F/\partial p_A^N}{\partial F/\partial \widetilde{x}^N} \quad \text{and} \quad \frac{\partial x_A^N}{\partial \mu_A^N} = -\frac{\partial F/\partial \mu_A^N}{\partial F/\partial \widetilde{x}^N}.$$

By replacing for these derivatives in the first-order conditions (4) and (5), and by imposing symmetry, we obtain the symmetric equilibrium levels of investment in capacity and the subscription fees, as reported in the Proposition.

Remark that if we set $K = 0$, we obtain the same equilibrium expressions as in our baseline model. In addition, note that K should not be too high (specifically, not higher than $a\lambda/2$) to obtain a positive level of entry by CPs.

Observe also that, whenever $K > 0$, $\overline{h}_K^N$ strictly increases with the advertising rate a. In words, the higher the advertising rate, the higher the number of CPs that enter the market.

II(ii). *Discrimination*

In the discriminatory regime, each ISP offers a priority lane and a non-priority lane to CPs. The CPs that opt for priority at ISP i pay a fixed fee f_i, whereas the non-priority lane is offered for free. In addition, the CPs incur a fixed cost K whenever they connect to an ISP (whatever the lane).

Stage 2: Content providers' and end users' decisions. At the second stage, each CP decides whether to multi home, to single home or to stay out of the market and, if it enters the market, whether to pay for priority. The CPs that are the most congestion-sensitive opt for the priority lane. A CP of type h connects to the priority lane at ISP i if $h \leq \overline{h}_i^D$, where $\overline{h}_i^D$ solves

$$a\lambda x_i^D(1 - \overline{h}_i^D w_i^P) - f_i - K = 0. \tag{6}$$

Furthermore, the CP of type $\widetilde{h}_i$ which is indifferent between the priority lane and the non-priority lane at ISP i is defined by

$$a\lambda x_i^D(1 - \widetilde{h}_i w_i^P) - f_i - K = a\lambda x_i^D(1 - \widetilde{h}_i w_i^{NP}) - K. \tag{7}$$

From (6) and (7), the total number of CPs that pay for priority at ISP i is $\max\{\overline{h}_i^D - \widetilde{h}_i, 0\}$. Equation (7) implies that$f_i = a\lambda x_i^D \widetilde{h}_i \left(w_i^{NP} - w_i^P\right)$, and replacing for this expression into (6), we obtain

$$a\lambda x_i^D \left[1 - \left(\left(\overline{h}_i^D - \widetilde{h}_i\right) w_i^P + \widetilde{h}_i w_i^{NP}\right)\right] - K = 0.$$

By dividing the latter expression by $\overline{h}_i^D$ and using w_i^D, we find that the type of the marginal CP that enters at ISP i is

$$\overline{h}_i^D = \mu_i^D \frac{a\lambda x_i^D - K}{\lambda x_i^D \left(a + a\lambda x_i^D - K\right)}. \tag{8}$$

The type of the marginal CP $\overline{h}_i^D$ is independent of the priority fee and takes an expression similar to the total number of CPs at ISP i in the net neutrality regime (which is given by (1)). As in the net neutrality regime, the demand and congestion effects are at play here, and the same reasoning applies to determine which effect dominates, according to the value of K. In addition, we have

$$\widetilde{h}_i = \frac{a\mu_i^D f_i}{\lambda x_i^D(a + a\lambda x_i^D - K)(a\lambda x_i^D - f_i - K)}. \tag{9}$$

Simultaneously, at stage 2, each consumer chooses whether to subscribe to ISP A or ISP B. The indifferent consumer $\widetilde{x}^D$ is given by

$$R+v\overline{h}_A^D+\frac{d}{w_A^D}-p_A^D-t\widetilde{x}^D=R+v\overline{h}_B^D+\frac{d}{w_B^D}-p_B^D-t\left(1-\widetilde{x}^D\right). \tag{10}$$

By replacing for $\overline{h}_A^D$ and $\overline{h}_B^D$ into (10), the indifferent consumer satisfies $F^D=0$, where

$$\begin{aligned} F^D \equiv\ & \frac{\mu_B^D\left(v-d\lambda\left(1-\widetilde{x}^D\right)\right)\left(a\lambda\left(1-\widetilde{x}^D\right)-K\right)}{\lambda\left(1-\widetilde{x}^D\right)\left(a+a\lambda\left(1-\widetilde{x}^D\right)-K\right)}- \\ & \frac{\mu_A^D\left(v-d\lambda\widetilde{x}^D\right)\left(a\lambda\widetilde{x}^D-K\right)}{\lambda\widetilde{x}^D\left(a+a\lambda\widetilde{x}^D-K\right)}+d\left(\mu_B^D-\mu_A^D\right)-t(1-2\widetilde{x}^D)-(p_B^D-p_A^D), \end{aligned} \tag{11}$$

and, therefore, we have $\widetilde{x}^D=\widetilde{x}^D(p_A^D,p_B^D,\mu_A^D,\mu_B^D)$. The number of users of ISP A and ISP B are then $x_A^D=\widetilde{x}^D$ and $x_B^D=1-\widetilde{x}^D$. Note that equation (11) for the discriminatory regime is similar to equation (3) for the net neutrality regime, and that $\widetilde{x}^D$ is independent of the priority fees.

Stage 1: ISPs' decisions. At the first stage, the two ISPs choose simultaneously their capacities, subscription fees and priority fees. The maximization problem of ISP i can be expressed as follows

$$\max_{p_i^D,\,\mu_i^D,\,f_i}\Pi_i^D=p_i^Dx_i^D+\left(\overline{h}_i^D-\widetilde{h}_i\right)f_i-C\left(\mu_i^D\right),$$

where $x_i^D=x_i^D(p_A^D,p_B^D,\mu_A^D,\mu_B^D)$. The corresponding first-order conditions are

$$\frac{\partial\Pi_i^D}{\partial p_i^D} = x_i^D+\left(p_i^D+\frac{\partial((\overline{h}_i^D-\widetilde{h}_i)f_i)}{\partial x_i^D}\right)\frac{\partial x_i^D}{\partial p_i^D}=0, \tag{12}$$

$$\frac{\partial\Pi_i^D}{\partial\mu_i^D} = \left(p_i^D+\frac{\partial((\overline{h}_i^D-\widetilde{h}_i)f_i)}{\partial x_i^D}\right)\frac{\partial x_i^D}{\partial\mu_i^D}-C'\left(\mu_i^D\right)+\frac{\partial((\overline{h}_i^D-\widetilde{h}_i)f_i)}{\partial\mu_i^D}=0, \tag{13}$$

$$\frac{\partial\Pi_i^D}{\partial f_i} = \frac{\partial((\overline{h}_i^D-\widetilde{h}_i)f_i)}{\partial f_i}=0. \tag{14}$$

We obtain the following result.

Proposition 2 Under discrimination, if the CPs incur a fixed cost K to connect at an ISP, in the symmetric equilibrium, the capacity level, the priority fee, the subscription fee, the number of CPs and the levels of congestion are given by:

$$\mu_K^D=(C')^{-1}\left(\frac{a(d+v)\lambda-2vK}{\lambda(a(2+\lambda)-2K)}+\frac{(a\lambda-2K)\left(a(4+\lambda)-2\left(K+\sqrt{2}\sqrt{a(a(2+\lambda)-2K)}\right)\right)}{\lambda(a(2+\lambda)-2K)}\right),$$

$$f_K=\frac{(a\lambda-2K)\left(a(2+\lambda)-2K-\sqrt{2}\sqrt{a(a(2+\lambda)-2K)}\right)}{2a(2+\lambda)-4K},$$

$$
\begin{aligned}
p_K^D \;=\; & t+\frac{4(-4vK(a-K+a\lambda)+a^2(d+v)\lambda^2)\mu_K^D}{\lambda(a(2+\lambda)-2K)^2} \\
& -\frac{2\mu_K^D\left(2K(4K^2-4aK(3+\lambda)+a^2(8+8\lambda+\lambda^2))-2a^3\lambda^2\right)}{\lambda(a(2+\lambda)-2K)^2} \\
& -\frac{2\mu_K^D\sqrt{2}\sqrt{a(a(2+\lambda)-2K)}(8K^2+a^2\lambda^2-2aK(4+3\lambda))}{\lambda(a(2+\lambda)-2K)^2},
\end{aligned}
$$

$$
\overline{h}_K^D=\frac{2\left(a\lambda-2K\right)\mu_K^D}{\lambda\left(a\left(2+\lambda\right)-2K\right)},\;\widetilde{h}_K=\frac{2\sqrt{2}\sqrt{a}\left(a\left(2+\lambda\right)-2K-\sqrt{2}\sqrt{a\left(a\left(\lambda+2\right)-2K\right)}\right)\mu_K^D}{\lambda\left(a\left(\lambda+2\right)-2K\right)^{\frac{3}{2}}},
$$

$$
w_K^P=\frac{\sqrt{\lambda+2-\frac{2K}{a}}}{\sqrt{2}\mu_K^D},\; w_K^{NP}=\frac{\sqrt{2}\left(\lambda+2-\frac{2K}{a}\right)^{\frac{3}{2}}}{4\mu_K^D},\; w_K^D=\frac{a\left(\lambda+2\right)-2K}{2a\mu_K^D}.
$$

Proof: We proceed as in the net neutrality regime, by applying the Implicit Function Theorem to (11) in order to determine the derivatives $\partial x_i^D/\partial p_i^D$ and $\partial x_i^D/\partial \mu_i^D$. We calculate

$$
\frac{\partial x_A^D}{\partial p_A^D}=-\frac{\partial F^D/\partial p_A^D}{\partial F^D/\partial \widetilde{x}^D}\text{ and }\frac{\partial x_A^D}{\partial \mu_A^D}=-\frac{\partial F^D/\partial \mu_A^D}{\partial F^D/\partial \widetilde{x}^D}.
$$

By replacing for these derivatives in the three first-order conditions, and by imposing symmetry, we obtain the symmetric equilibrium levels of investment in capacity, the subscription fees and the priority fees, as reported in the Proposition.

Again, if $K=0$, we obtain the same equilibrium expressions as in our baseline model.

II(iii). *Net neutrality vs. discrimination*

We can now compare the equilibrium in the two regimes, and prove that the investment in broadband capacity and CPs' entry are higher under the discriminatory regime than under the net neutrality regime. Since $C''>0$, $(C')^{-1}$ is an increasing function. Since, furthermore, the parenthesis in the right hand side of μ_K^D in Proposition 2 is higher than the parenthesis in the right hand side of μ_K^N in Proposition 1, we have $\mu_K^D\geq\mu_K^N$.

Indeed, the difference between the terms in parenthesis in the RHS of μ_K^D and μ_K^N is equal to

$$
\frac{(a\lambda-2K)}{\lambda\left[a(2+\lambda)-2K\right]}\left[a(4+\lambda)-2\left(K+\sqrt{2}\sqrt{a^2(2+\lambda)-2Ka}\right)\right].
$$

The first term of this expression is positive as we have assumed that $a\lambda>2K$. Let $\psi(K)=a(4+\lambda)-2\left(K+\sqrt{2}\sqrt{a^2(2+\lambda)-2Ka}\right)$. We find that

$$
\psi'(K)=-2\left[1-\frac{\sqrt{2}a}{\sqrt{a^2(2+\lambda)-2Ka}}\right]<0,
$$

as $a\lambda>2K$ implies that the term into brackets is positive. Since $\psi(K)$ is a decreasing function, its minimum is reached at the upper bound for K, i.e., $K=a\lambda/2$. Since $\psi(a\lambda/2)=0$,[2] for all $K\in[0,a\lambda/2]$, we have $\psi(K)\geq 0$, which proves that $\mu_K^D\geq\mu_K^N$. In turn, it implies that $\overline{h}_K^D\geq\overline{h}_K^N$.

[2] Note also that $\psi(0)>0$ for all $\lambda>0$.

III. UNIFORM ACCESS FEE

In this annex, we solve for the equilibrium when ISPs offer a single traffic lane, but can charge a uniform fee to access it. We use the superscript "U" to designate this scenario.

Stage 2: Content providers' and end users' decisions. Since there is a single traffic lane, the waiting times are determined in a similar way as in the net neutrality regime. Therefore, we have

$$w_i = \frac{1}{\mu_i - \lambda x_i^U \overline{h}_i^U}. \tag{15}$$

Let f_i denote the termination fee charged by ISP i to CPs. At the second stage, each CP decides whether to multi home, to single home or to stay out of the market. CP h's profit is given by

$$\Pi_h^U = \begin{cases} a\lambda x_A^U \left(1 - hw_A^U\right) - f_A + a\lambda x_B^U \left(1 - hw_B^U\right) - f_B & \text{it connects to both ISPs} \\ a\lambda x_i^U \left(1 - hw_i^U\right) - f_i & \text{if} \quad \text{it connects only to ISP } i \\ 0 & \text{otherwise.} \end{cases}$$

The CP of type h connects to ISP i iff $h \leq \overline{h}_i^U$, where $\overline{h}_i^U$ solves $a\lambda x_i^U (1 - \overline{h}_i^U w_i^U) - f_i = 0$. Using (15), we find that

$$\overline{h}_i^U = \mu_i^U \frac{a\lambda x_i^U - f_i}{\lambda x_i^U \left(a + a\lambda x_i^U - f_i\right)}.$$

Simultaneously, at Stage 2, each consumer chooses whether to subscribe to ISP A or ISP B. The indifferent consumer $\widetilde{x}^U$ is given by

$$R + v\overline{h}_A^U + \frac{d}{w_A^U} - p_A^U - t\widetilde{x}^U = R + v\overline{h}_B^U + \frac{d}{w_B^U} - p_B^U - t\left(1 - \widetilde{x}^U\right). \tag{16}$$

By replacing for $\overline{h}_A^U$ and $\overline{h}_B^U$ into (16), the indifferent consumer $\widetilde{x}^U$ satisfies $F^U = 0$, where

$$\begin{aligned} F^U \equiv & \frac{\mu_B^U \left(v - d\lambda\left(1 - \widetilde{x}^U\right)\right) \left(a\lambda\left(1 - \widetilde{x}^U\right) - f_B\right)}{\lambda\left(1 - \widetilde{x}^U\right)\left(a + a\lambda\left(1 - \widetilde{x}^U\right) - f_B\right)} \\ & - \frac{\mu_A^U \left(v - d\lambda\widetilde{x}^U\right)\left(a\lambda\widetilde{x}^U - f_A\right)}{\lambda\widetilde{x}^U\left(a + a\lambda\widetilde{x}^U - f_A\right)} + d\left(\mu_B^U - \mu_A^U\right) - t(1 - 2\widetilde{x}^U) - (p_B^U - p_A^U), \end{aligned}$$

and, therefore, we have $\widetilde{x}^U = \widetilde{x}^U(p_A^U, p_B^U, \mu_A^U, \mu_B^U, f_A, f_B)$. The number of users of ISP A and ISP B are then $x_A^U = \widetilde{x}^U$ and $x_B^U = 1 - \widetilde{x}^U$, respectively. Note that $\widetilde{x}^U$ depends on the (uniform) termination fees.

Stage 1: ISPs' decisions. At the first stage, the two ISPs choose simultaneously their capacities, subscription fees and uniform termination fees. The maximization problem of ISP i can be expressed as follows

$$\max_{p_i^U,\, \mu_i^U,\, f_i} \Pi_i^U = p_i^U x_i^U + \overline{h}_i^U f_i - C\left(\mu_i^U\right),$$

where $x_i^U = x_i^U(p_A^U, p_B^U, \mu_A^U, \mu_B^U, f_A, f_B)$. The three first-order conditions for profit maximization are

$$\frac{\partial \Pi_i^U}{\partial p_i^U} = x_i^U + \left(p_i^U + f_i \frac{\partial \overline{h}_i^U}{\partial x_i^U}\right)\frac{\partial x_i^U}{\partial p_i^U} = 0, \tag{17}$$

$$\frac{\partial \Pi_i^U}{\partial \mu_i^U} = \left(p_i^U + f_i \frac{\partial \overline{h}_i^U}{\partial x_i^U}\right)\frac{\partial x_i^U}{\partial \mu_i^U} - C'\left(\mu_i^U\right) + f_i \frac{\partial \overline{h}_i^U}{\partial \mu_i^U} = 0, \tag{18}$$

$$\frac{\partial \Pi_i^U}{\partial f_i} = \left(p_i^U + f_i \frac{\partial \overline{h}_i^U}{\partial x_i^U}\right)\frac{\partial x_i^U}{\partial f_i} + \overline{h}_i^U + f_i \frac{\partial \overline{h}_i^U}{\partial f_i} = 0. \tag{19}$$

We begin by solving for p_i^U in the first-order condition (17), which gives p_i^U as a function of f_i and μ_i^U. We then replace for this expression of p_i^U into the first-order condition (18), and solve for μ_i^U. We obtain that

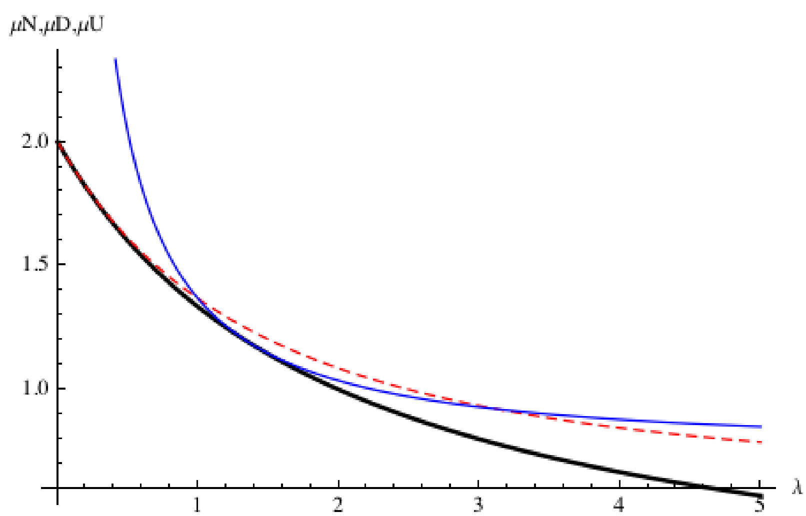
$$\mu_i^U\left(f_i\right) = \frac{\lambda a\left(d+v+2f_i\right) - 2f_i\left(v+2f_i\right)}{\lambda\left[\left(2+\lambda\right)a - 2f_i\right]}.$$

Note that $\mu_i^U(0) = (d+v)/\left(2+\lambda\right) \equiv \mu^N$. Furthermore, $\mu_i^U(f_i)$ increases with f_i (as $\partial\mu_i^U/\partial f_i > 0$ at $a = 0$ and $\partial\mu_i^U/\partial f_i$ increases with a). Therefore, $\mu_i^U \geq \mu^N$, with a strict inequality if $f_i > 0$. In other words, capacity investments are higher under this alternative net neutrality regime than under the net neutrality regime with no termination fee.

Finally, we replace for $\mu_i^U(f_i)$ into the last first-order condition, (19), and solve for the optimal f_i. This FOC has four roots, but only one satisfies the second-order conditions. In the symmetric equilibrium with the quadratic investment cost function, we finally obtain that

$$\mu^U = \frac{v + a\left(4+\lambda\right) - 2\sqrt{a\left(2v - d\lambda + 2a\left(2+\lambda\right)\right)}}{\lambda}.$$

The following figure compares the equilibrium capacity investments for quadratic investment costs under net neutrality (μ^N), discrimination (μ^D) and uniform pricing (μ^U), as a function of λ, for the following parameter values: $a = 1$, $t = 1$, $v = 3$, $d = 1$. As one can see, capacity investment under uniform pricing is always (at least weakly) higher compared to the net neutrality regime. However, it can be either higher (for low or high values of λ) or lower (for intermediate values of λ) than capacity investment under discrimination.



μ^N: thick black line, μ^D: red dashed line, μ^U: blue solid line